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Potentials of General-Purpose Reasoning Assistant System EUODHILOS

Every universe of discourse has its logical structure.
S. K. Langer (1925)

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Summary

Much work has been done on special-purpose reasoning assistant systems whose underlying logics are fixed. In contrast with such a trend, this paper is devoted to a new dimension of computer-assisted reasoning research, that is, a general-purpose reasoning assistant system that allows a user to define his or her own logical system relevant for the objects in the problem domain and to reason about them.

In the first half of the paper, the need, significance and design principle of EUODHILOS : a general-purpose system for computer-assisted reasoning, are discussed, then the system overview is described, placing emphases on the following three points : (1) formal system description language, (2) proving methodology based on several sheets for logical thought, (3) visual human-computer interface for reasoning. In the latter half, the potentials and usefulness of EUODHILOS are demonstrated through experiments and experiences of its use by a number of logics and proof examples therein, which have been used or devised in computer science, artificial intelligence and so on.

INTRODUCTION

A new dimension of computer-assisted reasoning research is being explored in this paper. It aims at a general-purpose reasoning assistant system that allows a user to interactively define the syntax and inference rules of a formal system and to construct proofs in the defined system. We have named such a system EUODHILOS, an acronym reflecting our philosophy or observation that *every universe of discourse has its logical structure*, which turns out to spell and sound like a Greek philosopher's name.

In these days, various logics play important and even essential roles in computer science and artificial intelligence (e.g., [Turner 84], [Genesereth 87], [Smets 88], [Thistlewaite 88]),

and surprisingly in aesthetics which is thought of as being in a directly opposite position to logic (e.g., [Langer 25], [Kunst 76], [Rahn 79]). Specifically, it can be said that they provide expressive devices for objects and their properties, and inference capabilities for reasoning about them. It is also the case that symbols manipulating methods provided in logics are basically common to all scientific activities. So far, they have made use of a wide variety of logics, including first-order, higher-order, equational, temporal, modal, intuitionistic, relevant, type theoretic logics and so on. However, implementing an interactive system for developing proofs is a daunting and laborious task for any style of presentation of these logics. For example, one must implement a parser, term and formula manipulation operations (such as substitution, replacement, juxtaposition, etc.), definitions, inference rules, rewriting rules, proofs, proof strategies and so on, depending on the logic to be needed. Thus, it is desirable to find a general theory of logics and a general-purpose reasoning assistant system that captures the uniformities of a large class of logics so that much of this effort can be expended once and for all. This is completely the same observation and motivation as in [Griffin 87]. We aim at building an easy to use and general reasoning system which handles as many of these logics as possible.

There are three major subjects to be pursued for such an interactive and general reasoning support system. One is, of cource, a language expressive enough to describe a large class of logics. The second is the kind of reasoning styles suitable for human reasoners which should be taken into account. More generally, reasoning (proving) methodology, which reminds us of programming methodology, needs to be investigated. The third subject is reasoning-oriented human-computer interface that may be well established as an aspect of reasoning supporting facilities. An easy to use system with good interface would be helpful for one to conceive ideas in reasoning and promote them further.

We believe that a general-purpose reasoning assistant system incorporating these points should cater to the mathematician or programmer who wants to do proofs, and also to the logician or computer theorist who wants to experiment with different logical systems according to the respective problem domains.

The remainder of this paper is organized as follows. In the first half of the paper, following the discussion of the need, significance and design philosophy of EUODHILOS, a system summary of EUODHILOS under development is described, where we place emphases on the following three points : (1) formal system description language, (2) proving methodology based on several sheets for logical thought, (3) visual human-computer interface for reasoning. In the latter half, the potentials and usefulness of EUODHILOS are shown through experiments and experiences of its use by a number of logics and proof examples therein, which have been used or devised in computer science, artificial intelligence and so on. They includes a logical puzzle, an inductive proof and the halting problem with first-order logic, second-order logic, propositional modal logic, intuitionistic type theory, program verification with Hoare logic and dynamic logic, and a reflective proof and Montague's semantics with intensional logic.

NEED, SIGNIFICANCE AND DESIGN PHILOSOPHY

Much work has been devoted to special-purpose reasoning assistant systems whose underlying logics are fixed (e.g., [Gordon 79], [Weyhraich 80], [Constable 82], [Ketonen 84], [Trybulec 85]). At this stage, why are we about to pursue or explore a new dimension such as a general-purpose reasoning assistant system ?

We first take up some issues concerned with the generality in reasoning assistant system and several aspects to view such a generality. We have already found and recognized that in these days a logic or logical methodology is forming a kind of paradigm for promoting computer science, artificial intelligence and so on. And we stated that it is desirable to find a general theory of logics and a general-purpose reasoning assistant system that captures the uniformities of a large class of logics so that much effort for providing reasoning facilities can be expended once and for all, and hence we aim at building an easy to use and general reasoning system which handles as many of these logics as possible. This was our first motivation for pursuing the generality in reasoning assistant system. The second issue comes from the rigorous approach to program construction. Abrial [Abrial 84] claims that a general-purpose proof checker be perhaps one of a set of tools for computer aided programming when we consider program construction from various theories. We are certainly in a situation that before embarking on the construction of a program we need to study its underlying theory, that is to give a number of definitions, axioms and theorems which are relevant to the problem at hand. The third issue is concerned with the construction of a logical model, or more generally methodology of science. We observe that the whole phases of human reasoning process consist of the following three phases : (1) making mental images about the objects or concepts, (2) making logical models which describe the mental images, (3) examining the models to make sure that they coincide with mental images. It is not conceivable that the phase (1) could be aided mechanically since some part of the phase (1) is very creative. On the other hand, it is very likely that the phases (2) and (3) are largely supported mechanically by allowing to modify or revise the definition of the language used for the modeling and by introducing certain reasoning devices. These are just the points that a general-purpose reasoning assistant system is intended to support. A philosophical aspect of the generality from a logical point of view can be found in [Langer 25] and Wittgenstein's philosophy. Langer stated that "every universe of discourse has its logical structure". It eventually supports our discussions about the need and significance of the generality in reasoning assistant system from the philosophical point of view.

Taking these needs, observations and philosophy into consideration, the fundamental design principles of EUODHILOS are set up as follows :

- Realization of a general reasoning sytem, reflecting the philosophy that every universe of discourse has its logical structure.
- Support of logical thought, symbolic or logical manipulations done by human reasoners
- Provision of an easy to use environment for supporting proof constructions

- Environment for experimenting logical model construction and methodology of science

which lead us to the research and development of general-purpose reasoning assistant system EUODHILOS with the following outstanding features :

- Formal system description language based on the definite clause grammar (DCG)
- Proving methodology using sheets of thought
- Reasoning-oriented human-computer interface

In what follows, we will sketch each of these issues in more detail .

OVERVIEW OF EUODHILOS

Functional features

We list the main features of EUODHILOS and explain them briefly (see [Satoh 88a] and [Satoh 88b] for the details). We start by describing the language of a logic to EUODHILOS. Fundamentally, EUODHILOS has almost no defaults, so it must be told everything,

Formal system description language

What on earth is a logic ? Or what language should be expressive enough to describe or deal with logics ? The answers to these questions, in general, could turn out to settle the formal system description language for capturing the uniformities of a large class of logics so that it can be used as the basis for implementing proof systems. There have been some attempts to pursue it, sharing a common goal with our EUODHILOS (e.g., Prolog is employed as a logic description language in [Sawamura 86], λProlog in [Felty 88] and [Miller 87], Martin-Löf's intuitionistic type theory in [Harper 87] and [Griffin 87], a specification language for a wide variety of logics in [Abrial 84], an attribute grammar formalism in [Reps 84] and a metalanguage ML in [Gordon 82]). McRobbie ([McRobbie 88]) communicated to us his approach to the construction of a general purpose theorem prover for non-classical logics.

A contemporary logic, in general, may be considered as having a logical framework consisting of proof theory and model theory. Proof theory is to specify a syntactical part of logic and model theory a semantical part of logic. In this paper we are mainly concerned with specifying the syntactical aspect of logic. From a syntactic point of view, a formal system (logical system), in general, is supposed to be specified by the two constituents : Language system and derivation system.

(1) Language system

A language is a tool for talking about objects and is formed from underlying primitive symbols. It is usually specified by utilizing some of the following items : variables, constants and functions as individual symbols, predicates (including equality), logical connectives, auxiliary symbols. The attributes such as type, sort, arity, operator precedence are sometimes associated with some of these symbols. Once these primitive symbols are specified, complexies

such as terms, formulas, etc., are constructed from them by fomation rule. Besides, mechanisms for defining or abbreviating symbols, are usually required. One of the main questions that may be raised at this point is the following : what kind of metalanguage is natural and sufficient enough to describe such an object language ?

(2) Derivation system

Derivation system gives us a means to manipulate logical languages, which are specified by axioms, inference rules, derived rules, rewriting rules, and concepts of proofs, etc. Insofar as we are confined ourself to the existing types of formal system, we can enumerate primitive operations included in them, for instance, substitution, replacement, juxtaposition, detachment, renaming, unification, instantiation, etc. are common operations among various logics except for the differences of languages. Since we consider a general-purpose reasoning system for logics, we have to provide a general method for such symbol manipulations. So, one of the main questions that may be raised at this point is the following : what sort of primitive operations and constraints on objects should be sufficient to manipulate logics and how they can be provided in a generic manner ?

In addition to them, we would need to pay attention to the following concepts proper in logics, such as "free" and "bound", "something is free for a variable in an expression", etc., although these can be often dealt with in a recursive fashion. In what follows, we will give a partial but feasible answer to specifying logics.

Specifying a logical syntax and expressiveness of the definite clause grammar

In EUODHILOS, a language system to be used is designed and defined by a user, a current solution for formal system description language is to employ so called definite clause grammar formalism (DCG) [Pereira 80], where the problem of recognizing, or parsing a string of a language is then transformed into the problem of proving that a certain theorem follows from the definite clause axioms which describe the language. The DCG formalism for grammars is a natural extension of context-free grammar (CFG). As such, DCG inherits the properties which makes CFG so important for language theory such as the modularity of a grammar description and the recursive embedding of phrases which is characteristic of almost all interesting languages, including languages of logics. It is, however, well known that CFG is not fully adequate for describing natural language, nor even many artificial languages. DCG overcomes this inadequacy by extending CFG in the following three points [Pereira 80].

(i) context-dependency

(ii) parameterized nonterminal

(iii) procedure attachment

These also yield great advantages for specifying logical grammars, compared with those mentioned above. DCG provides for context-dependency in a grammar, so that the permissible forms for a phrase may depend on the context in which that phrase occurs in the string. DCG is somewhat similar to attribute grammar in the sense that context free grammar is made context

sensitive by associating with grammar rules a semantical facility [Reps 84]. The necessity of context-dependency is often encountered in defining logical syntax. The following examples show how naturally and economically DCG allows us to express context-dependency occurring in the ordinary logical practice and allows arbitray tree structure to be build in the course of the parsing, with the help of (ii) and (iii).

Let us describe some concrete examples of the syntax definition in order to see the paradigm of definite clause grammar formalism. The defining clause of first-order terms such as "If f is a function symbol of arity 2 and t and s are terms, then f(t, s) is a term" is represented as

term(f(T,S)) --> functor (f), "(", term(T), ",", term(S), ")", {arity(f, 2)}.

The defining clause of terms in intensional logic such as "If A is a term of type (a, b) and B a term of type a, then A•B is a term of type b" is represented as

term(A•B, b) --> term(A, (a,b)), "•", term(B, b).

Once a definite clause definition for a logical syntax have been given, then the bottom-up parser [Matsumoto 83] and unparser for the defined language are automatically generated, which are to be internally used in all the phases of symbol manipulations. The reason why we do not generate a top-down parser for the defined language is as usual to avoid the anomaly of left-recursiveness which often appears in the ordinary definition of a logical syntax. The internal structures of the expressions of the language are automatically constructed as well just at the same time as the automatic generation of the parser and unparser. These functions greatly lighten a user's burden in setting up his own language. (The details of the methods are presented in [Satoh 88].)

Specifying a derivation system

A derivation system consists of an inference system and a rewriting system. They are given in a natural deduction style presentation [Prawitz 65] by a user. Especially, an inference rule is stated as a triple consisting of three elements, where the first is the derivations of the premises of a rule, the second the conclusion of a rule, and finally the third the restrictions that are imposed on the derivations of the premises, such as variable occurrence condition (eigenvariable) and substitutability such as "t is free for x in P". Well-known typical styles of logic presentations such as Hilbert's style, Gentzen's style, Equational style could be treated within this framework.

Inference rules are presented in terms of the schematic rule description language in a natural deduction style as follows :

$$\frac{\begin{array}{cccc}[\text{Assumption}_1] & [\text{Assumption}_2] & \cdots & [\text{Assumption}_n] \\ \vdots & \vdots & & \vdots \\ \text{Premise}_1 & \text{Premise}_2 & & \text{Premise}_n\end{array}}{\text{Conclusion}}$$

where brakets are used to encompass a temporary assumption to be discharged, ":" denotes a sequence or a subtree of formulas which is a part of a proof from the assumption and each assumption is optional. If a premise has the assumption, its subtree of a proof indicates a conditional derivation. In a forward reasoning, an inference rule may be permitted to apply if all the premises are obtained in this manner and the application condition is satisfied. Then, the dependency of a conclusion on temporary assumptions is automatically calculated by the ordinary method [Ketonen 84]. In a backward reasoning, discharging the asumptions, generating some assumptions and checking the application conditions are in general impossible and hence delayed until completing the partial proof tree under construction. The definitions of derived rules are also permitted if they are justified for validity on a sheet of thought described below. They are convenient to shorten the lengthy and tedious derivation steps to some extent.

Similarly, rewriting rules are presented in the following schematic format :

$$\frac{exp_1}{exp_2}$$

where exp_1 and exp_2 denote the subexpresions occurring in the expressions before and after rewriting an expression respectively.

Proof construction facilities

The major drawback of reasoning in formal logic is that derivations tend to be lengthy and tedious because of the detailed level of derivations to be required in reasoning. Furthermore, performing formal derivations is time-consuming and error-prone. The readers may notice that such a situation is quite similar to the one in the formal development of programs in which programs can be derived or transformed and properties of programs can be established. Using computers for formal reasoning is expected to overcome the problems with errors and the time-consuming task. The current version of EUODHILOS has the following unique facilities which support natural and efficient constructions of proofs in the defined formal system.

(1) Sheets of thought (or proof sheet)

This originated from a metaphor of work or calculation sheet and is apparently analogous to the concept of sheet of assertion which is due to C. S. Peirce [Peirce 74]. A sheet of thought, in our case, is supposed to be a field of thought where we are allowed to draft a proof, to compose proof fragments or detach a proof, or to reason using lemmas, etc., while a sheet of assertion is a field of thought where existential graph as an icon of thought is supposed to be drawn. Obviously, proving by the use of sheets of thought yields proof modularization considered to be useful for proving in large, which is analogous to the concept of program modularization, to borrow the term of software engineering. Technically, a sheet of thought is a window with multi-functions for reasoning in the multi-window environment of a Personal Sequential Inference machine (PSI).

(2) Tree-form proof

As mentioned above, inference and rewriting rules are presented in a natural deduction style. This naturally induces a construction of a proof into a tree-form proof with a justification for each line (node) indicated in the right margin. Consequentlly it leads to representing a proof structure explicitly, in other words, proof visualization.

(3) Proving methodology

It is desirable that reasoning or proof construction can be done along the natural way of thinking of human reasoners. Therefore EUODHILOS supports the typical method for reasoning, that is, top-down reasoning (backward reasoning), bottom-up reasoning (forward reasoning) and reasoning in a mixture of them. They are accomplished interactively on several sheets of thought. EUODHILOS also allows us to construct an abstract proof in the sense that metavariables ranging over syntactic domains of an object logic are permitted to occur in the process of the proof, that is, we can make a partially instantiated proof. Such a proving facility is very convenient for having an indeterminate or unknown predicate (such as invariant assertion in Hoare logic) unspecified temporarily in the proof constructing process.

It is planed to incorporate not only such a proving methodology but also methodology of science (e.g., Lakatos' mathematical philosophy of science [Lakatos 76], Kitagawa's relativistic logic of mutual specification [Kitagawa 63], etc.).

As an example of deduction process on sheets of thought, let us illustrate how one can proceed a deduction by using connection and separation functions of sheets of thought. In order to deduce forward by applying an inference rule, one has to start by selecting the formulas used as premises of the rule. Then one may select an appropriate inference rule from the rule menu which has been automatically generated at the time of logic definition, or he may input a formula as the conclusion. If one selects a rule, then the system applies the rule to the premises and derive the conclusion. If he gives the conclusion, then the system searches the rules and tries to find one which coincides with this deduction. In the case of backward reasonig, the reasoning process is converse to the forward reasoning, so that the intermediate proof may turn out to be separated into partially justified proof fragments and the complete justificiation of those partilly justified proof fragments is delayed to the completion of a final proof tree.

(i) Connection

(a) Connection by complete matching : Two proof fragments can be connected through a commom formula occurring in them when one of them is a hypothesis and the other a conclusion. The process begins by selecting the two formulas and invoking the proper operations. As a result, the proof fragments are connected into the one proof fragment. Schematically, This amounts to attaining the following inference figure which can be viewed as valid :

$\Gamma \vdash C$ (on a sheet of thought)
$\Delta, C, \Sigma \vdash A$ (on a sheet of thought)

$\Gamma, \Delta, \Sigma \vdash A$ (on a sheet of thought)

where Γ, Δ and Σ represent sequences of formulas (possibly empty), and A and C denote formulas in some defined logical system.

(b) Connection by the use of a rule of inference : This is essentially a forward reasoning and may be called a distributed forward reasoning. The process is similar to the above except that the connection is done from the distributed proof fragments through an appropriate rule of inference. Let us take an example schema of modus ponens :

$\Gamma \vdash A \supset B$ (on a sheet of thought)
$\Delta \vdash A$ (on a sheet of thought)

$\Gamma, \Delta \vdash A$ (on a sheet of thought)

with the same proviso, adding that B represents a formula. Besides, connection methods such as analogical matching, instantiation, etc., would beome extremely beneficial to intelligent reasoning system, which are left as future subjects.

(ii) Separation

The separation is the converse to the connection by complete matching. The separation process begins by selecting a formula occurring in a sheet of thought and invoking the proper operations. As a result, the proof fragment are detached into the two fragments. Schematically, This amounts to the converse to the connection by complete matching above. So we omit it.

Human-computer interface for reasoning

In order to make the system user-friendly and easy to use, we have paid much attention to the visualization of interface. In EUODHILOS the following facilities are available as human-computer interface for ease in communicating and reasoning with a computer, in particular facilities for inputting formulas and formula visualization.

(1) Formula editor

This is a structure editor for logical formulas and makes it easy to input, modify and display complicated formulas. In addition to ordinary editing functions, it provides some proper functions for formulas such as rewriting functions.

(2) Software keyboard and Font editor

These are used to make and input special symbols often appearing in various formal systems. It is a mater of course that provision of special symbol which reasoners are accustomed to use makes it possible to reason as usual on a computer.

(3) Stationery for reasoning

Independently of a logic under consideration, various reasoning tools such as decision procedures become helpful and useful in reasoning processes. In a sense it may also play a role of a model which makes up for a semantical aspect of reasoning. Currently, a calculator for Boolean logic is realized as a desk accessory.

The screen layout in Appendix 1 shows a proof in which formula editor and software keyboard are being used.

Implementation

Exploiting the bit-map display with multi-window environment, mouse, icon, pop-up-menu, etc., EUODHILOS is implemented in ESP language (an object-oriented Prolog) on PSI-II/SIMPOS, and its current size is about 5MB. The system configuration of EUODHILOS is illustrated in Appendix 2. The system consists of two major parts ; one for defining a user's logical system and the other for constructing proofs on sheets of thought.

EXPERIMENTS AND EXPERIENCES WITH EUODHILOS

We have tried to apply EUODHILOS to various types of reasoning. Logics and proof examples therein that we have dealt with so far on EUODHILOS include various pure logical formulas, the unsolvability of the halting problem and an inductive proof with first-order logic (NK), the equivalence between the principle of mathematical induction and the principle of complete induction with second-order logic, modal reasoning about programs with propositional modal logic (T), the reflective proof of a metatheorem and Montague's semantics of natural language with intensional Logic (IL), Martin-Löf's intuitionistic type theory, reasoning about program properties with Hoare logic and dynamic logic. These logics constitute a currently well-known and wide range of logics or formal systems.

In the former part of this section, taking up typical formal systems from various fields, we demonstrate how EUODHILOS can be used to specify a logic and construct a proof under the specified logic, together with brief annotations. In the latter part, we list some other proof experiments with different logics. The important point here is not the complexity of the examples, but rather the holistic understanding of a whole story played with EUODHILOS. These proof experiments with different logical systems could help to convince the readers of the potential and usefulness of EUODHILOS in a much wider range of applications.

Martin-Löf's intuitionistic type theory and a constructive proof

The first reasoning system we have chosen as an example is a tiny subset of constructive type theory described in [Martin-Löf 84] and [Backhouse 88]. The principal expression in intuitionistic type theory is a judgement of the form "$a \in p$", reads "a is a proof of a proposition " in one interpretation, where "a" is an expression in λ-calculus and "p" is a first-order formula

interpreted as a type. The judgement is naturally and well described in the framework of DCG. Intuitionistic type theory is defined by a number of natural deduction style inference rules [Martin-Löf 84] which are of course best suited to our treatment of rules. In Appendix 2, the screen layout of the proof of the theorem ~~(P ∨ ~P) is shown, which means that the law of excluded middle cannot be refuted, and is an instance of Glivenko's theorem that if P is any tautology of the classical propositional calculus then the proposition ~~P is always constructively valid.

First we set up the language for proving the theorem first in terms of BNF for reference and then its DCG description.

Tiny language for type theory (BNF)

<judgement> ::= <term> ∈ <type>
<term> ::= <variable> | <constant> | ~<term> |<function>•<term> | λ<variable>.<term> |
inl(<term>) | inr(<term>) | <meta-term>
<variable> ::= x
<constant> ::= a | b
<function> ::= f

<type> ::= <basic-type> | <type> ∨ <type> | <type> ⊃ <type> | <meta-type>
<basic-type> ::= P | ⊥
<meta-term> ::= F
<meta-type> ::= A | B

Tiny language for type theory (DCG)

The following DCG definition may be somewhat tedious and roundabout for the reasons of the abilities of the current bottom-up parser and unparser generators. For exampe, the production rule "judgement --> term, ∈, type" have to be described in terms of the two clauses as follows:

judgement ---> term, in, type ;
in --> "∈" ;

and the terminal "∈" have to be declared as an operator. It, however, will be easily improved so as to be more natural in the next version of EUODHILOS, making it possible to use the definite clause grammar augmented by adding operator precedence.

Syntax of object langauge :

judgement ---> term, in, type ;
in --> "∈" ;
term --> lambda, variable, ".", term1 ;
lambda --> "λ" ;
term --> term1 ;
term1 --> term1, apply, term2 ;

apply --> "•" ;
term1 --> term2 ;
term2 --> "(", term, ")" ;

term2 --> or-intro, "(", term, ")" ;

or-intro --> "inr" | "inl" ;
term2 --> variable ;
term2 --> constant ;

variable --> x | f | a | b ;
type --> type1, imply, type ;

imply --> "⊃" ;
type --> type1 ;
type1 --> type2, or, type1 ;

or --> "∨" ;
type1 --> type3 ;
type3 --> "(", type, ")" ;

type3 --> not, type3 ;

not --> "~" ;
type3 --> basic-type ;
basic-type --> "P" | "⊥" ;

Syntax of meta language :
term1 --> meta-term, "(", meta-var, ")" ;
term2 --> meta-term ;

meta-term --> "F" ;

type3 --> meta-type ;

meta-type --> "A" | "B".

Note that the syntax definition for meta language is needed for defining inference rules schematically.

Inference Rules

Intuitionistic type theory is defined by a number of natural deduction style inference rules [Martin-Löf 84]. For our purpose of illustration we consider just four rules and one rewrite rule. These are the rules for function introduction and elimination, the two rules for ∨ introduction, and the rewrite rule in lieu of the definition ~A = A ⊃ ⊥.

$$\frac{\begin{array}{c}[x \in A]\\ \vdots \\ F(x) \in B\end{array}}{\lambda x.\,(x) \in A \supset B} \quad (\lambda\text{-introduction } (\lambda\text{-I}))$$

$$\frac{a \in A \qquad f \in A \supset B}{\qquad\qquad} \quad (\supset\text{-elimination } (\supset\text{-E}))$$

$$\frac{f \bullet a \in B}{a \in A \quad \text{---------- (inl-introduction (inl-I))} \quad inl(a) \in A \vee B}$$

$$\frac{b \in B}{\text{---------- (inr-introduction (inr-I))} \quad inr(b) \in A \vee B}$$

$$\frac{A \supset \bot}{\text{---------- (definition as rewrite rule)} \quad \sim A}$$

We have specified both the language system and derivation system possibly sufficient to the proof below. We may often want ro revise or modify the defined logical system, due to the inconveniences encountered later. By the inconveniences, we mean the logical system is too weak, strong, redundant, or irrelevent to deal with the objects under consideration. Once a logical system has been specified, the revision or modification of it is critical and carefully must be done since the already established facts may not be guaranteed to hold. The current version of EUODHILOS have not supported such a theory revision yet. Note that it is always safe in case that the logical system is augmented by adding symbols, axioms and inference rules to the old system as far as the addition is consistent with the old one.

Proof

The manual proof of a tree form is described below and the overall screen layout is shown in Appendix 3.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[x \in P]^1}{inl(x) \in P \vee (P \supset \bot)}\,(\text{inl-I}\ \{1\}) \quad [f \in (P \vee (P \supset \bot)) \supset \bot]^2}{f \bullet inl(x) \in \bot}\,(\supset\text{-E}\ \{2\,1\})}{\lambda x.\, f \bullet inl(x) \in P \supset \bot}\,(\lambda\text{-I}\ \{2\})}{inr(\lambda x.\, f \bullet inl(x)) \in P \vee (P \supset \bot)}\,(\text{inr-I}\ \{2\}) \quad [f \in (P \vee (P \supset \bot)) \supset \bot]^2}{f \bullet inr(\lambda x.\, f \bullet inl(x)) \in \bot}\,(\supset\text{-E}\ \{2\})}{\lambda f.\, f \bullet inr(\lambda x.\, f \bullet inl(x)) \in (P \vee (P \supset \bot) \supset \bot) \supset \bot}\,(\lambda\text{-I}\ \{\})}{\lambda f.\, f \bullet inr(\lambda x.\, f \bullet inl(x)) \in \sim\sim(P \vee \sim P)}\,(\text{def}\ \{\})$$

where the justifications of the form (a rule name {dependencies}) are indicated in the right margin of the proof tree.

Hoare logic and program verification

Hoare logic [Hoare 69] is the most well known logic for the axiomatic semantics of a programming language and the verification of a program. Here we exemplify how such a notationally complicated formal logic can be easily dealt with on EUODHILOS. The principal formula in Hoare logic is a form of P{S}Q, reads "if P holds, then after executing the program S, Q holds", where P and Q are first-order formulas and S is a program in an ALGOL-like programming language. These syntactic objects are easily described in the framework of DCG, as well as the inference rules for Hoare logic. The screen layout of the correctness proof of a factorial program with the precondition "true" and the postcondition "z = x!" is shown in Appendix 4 .

Syntax definition in terms of DCG :

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h-formula --> formula, left-brace, program, right-brace, formula ;
left-brace --> "{" ;
right-brace --> "}" ;
formula --> formula, imply, formula1 ;
imply --> "⊃" ;
formula --> formula1 ;
formula1 --> formula1, or, formula2;
or --> "∨" ;
formula1 --> formula2 ;
formula2 --> formula2, and, formula3 ;
and --> "∧" ;
formula2 --> formula3 ;
formula3 --> "(", formula, ")" ;
formula3 --> not, formula3 ;
not --> "~" ;
formula3 --> "true" ;
formula3 --> term, equal,  term ;
equal --> " =" ;
formula3 --> meta-formula, "(", meta-term, ")" ;
formula3 --> meta-formula ;
term --> variable ;
term --> constant ;
term --> term, plus, term ;
plus --> "+" ;
term --> term, multiply, term ;
multiply --> "×" ;
term --> term, factorial ;
factorial --> "!" ;
term --> meta-term ;
variable --> "x" | "y" |"z" | meta-var ;
```

constant --> "1" | "0" ;
program --> assignment-statement ;
program, sequence,, program ;
while, formula, "do", program, "od" ;
if, formula, "then", program, "else", program, "fi" ;
meta-program ;
assignment-statement --> variable, assignment, term ;
sequence --> ";" ;
while --> "while" ;
if --> "if" ;
assignment --> ":=" ;
meta-program --> "A" | "B" ;
meta-var --> "X" ;
meta-term --> "T" ;
meta-formula --> "P" | "E" | "F" |"G".

Axioms and Theorems

(1) $E \wedge F \supset E$ (Conjuction-elimination)
(2) $P(X) \wedge X = T \supset P(T)$ (Substitution)
(3) $P(T)\ \{X := T\}\ P(X)$ (Assignment axiom)
(4) $true \supset 1 = 0!$ (Arithmetic)

Rewriting rule

$$\frac{z = y!}{z \times (y+1) = (y+1)!}$$ (Arithmetic rule)

Inference Rules

$$\frac{E \supset F \quad F\{A\}G}{E\{A\}G}$$ (Consequence rule 1)

$$\frac{E\{A\}F \quad F \supset G}{E\{A\}G}$$ (Consequence rule 2)

$$E\{A\}F \quad F\{B\}G$$

---------- (Composition rule)
E{A ; B} G

E ∧ F{A}G E ∧ ~F{B}G
---------- (Conditional rule)
E{if F then A else B fi}G

F ∧ G{A}F
---------- (Repetition rule)
F{while G do A od}F ∧ ~G

In Appendix 4, we show the screen layout of the correctness proof of a factorial program with the precondition "true" and postcondition "z = x!".

Dynamic logic and reasoning about programs

Dynamic logic [Harel 84] is a kind of multi-modal logic which is an extension to classical logic. The principal formulas in dynamic logic are the dynamic formulas of the form [a]p and the dual <a>p, read informally "after executing the program a the proposition p holds", where a is a regular or context-free program and p is a first-order or dynamic formula. They can be easily dealt with in the framework of DCG as follows.

Syntax definition in terms of DCG

```
formula-1 --> left-diamond, regular-program, right-diamond, formula-1 ;
left-diamond --> "<" ;
right-diamond --> ">" ;
formula-1 --> left-box, regular-program, right-box, formula-1 ;
left-box --> "[" ;
right-box --> "]" ;
formula-1 --> formula ;
formula --> formula, equivalence, formula0 ;

equivalence --> "≡" ;
formula --> formula0 ;

formula0 --> formula0, imply, formula1 ;

imply --> "⊃" ;
formula0 --> formula1 ;

formula1 --> formula1, or, formula2;

or --> "∨" ;
formula1 --> formula2 ;

formula2 --> formula2, and, formula3 ;

and --> "∧" ;
```

formula2 --> formula3 ;

formula3 --> "(", formula, ")" ;

formula3 --> not, formula3 ;

not --> "~" ;

formula3 --> "true" ;

formula3 --> term, equality, term ;

equality --> " =" ;

formula3 --> term, greater, term ;

greater --> ">" ;

formula3 --> term, greater-or-equal, term ;

greater-or-equal --> "≥" ;

formula3 --> meta-formula, "(", term, ")" ;

formula3 --> meta-formula ;

term --> variable ;

term --> constant ;

term --> term, plus, term ;

plus --> "+" ;

term --> term, minus, term ;

minus --> "-" ;

term --> term, multiply, term ;

multiply --> "x" ;

term --> term, factorial ;

factorial --> "!" ;

term --> meta-term ;

variable --> "x" | "y" |"z" | "n" | meta-var ;

constant --> "1" | "0" ;

regular-program --> assignment-statement ;

regular-program --> formula, guard ;

guard --> "?" ;

regular-program --> regular-program, sequence, regular-program ;

sequence --> ";" ;

regular-program --> regular-program,nondeterministic-selection, regular-program

nondeterministic-selection --> "|" ;

regular-program --> regular-program, unbounded-repetition ;

unbounded-repetition --> "*" ;

regular-program --> meta-program ;

assignment-statement --> variable, assignment, term ;

asignment --> ":=" ;

meta-program --> "A" | "B" ;

meta-var --> "X" ;
meta-term --> "T" ;
meta-formula --> "P" | "Q" | "R" | "S".

Axioms and Theorems

(1) $[Q?]P \equiv (Q \supset P)$ (test)
(2) $[X := T]P(X) \equiv P(T)$ (assignment axiom)
(3) $[A ; B]P \equiv [A][B]P$ (composition)
(4) $\langle A ; B\rangle P \equiv \langle A\rangle\langle B\rangle P$ (composition)
(5) $[A \mid B]P \equiv ([A]P \wedge [B]P)$ (nondeterministic selection)
(6) $P(X) \wedge X = T \supset P(T)$ (substitution)

(7) $x = 0 \supset (x = 0 \supset \text{true})$ (arith)
(8) $\langle(x = 0)?\rangle\text{true} \equiv (x = 0 \supset \text{true})$ (theorem)
(9) $n \geq 0 \wedge x = n+1 \supset \langle(x>0)?\rangle(x = n+1)$ (theorem)
(10) $x = n+1 \supset \langle z := x \times z\rangle(x = n+1)$ (theorem)
(11) $x = n+1 \supset \langle x := x - 1\rangle(x = n)$ (theorem)
(12) $z \times x! = n! \wedge x > 0 \supset [z := x \times z]\,(z \times (x-1)! = n!)$ (theorem)
(13) $z \times (x-1)! = n! \supset [x := x - 1]\,(z \times x! = n!)$ (theorem)
(14) $x = n \supset [z := 1](z \times x! = n!)$ (theorem)
(15) $z \times x! = n! \supset [(x = 0)?](z = n!)$ (theorem)

Rewriting rule

$$\frac{[A]P}{\sim\langle A\rangle\sim P}\ \text{(def)}$$

$$\frac{\sim P \equiv \sim Q}{P \equiv Q}\ \text{(neg-elim)}$$

$$\frac{\sim\sim P}{P}\ \text{(double-neg-elim)}$$

$$\frac{n \geq 0 \wedge x = n}{x \geq 0}\ \text{(arithmetic)}$$

$$\text{true} \wedge P$$

---------- (true-elim)

P

Inference Rules

P P ⊃ Q
---------- (modus ponens)
Q

P ⊃ Q
---------- (necessitation)
[A]P ⊃ [A]Q

P ⊃ [A]P
---------- (invariance)
P ⊃ [A*]P

n≥0 ∧ P(n+1) ⊃ <A>P(n)
---------- (convergence)
n≥0 ∧ P(n) ⊃ <A*>P(0)

P ⊃ <A>Q Q ⊃ <B>R
---------- (composition 1)
P ⊃ <A ; B>R

P ⊃ [A]Q Q ⊃ [B]R
---------- (composition 2)
P ⊃ [A ; B]R

P ⊃ <A>Q R ⊃ [A]S
---------- (derived-rule1)
P ∧ R ⊃ <A>(Q ∧ R)

P(Q) Q ≡ R
---------- (replacement 1)
P(R)

P ⊃ Q Q ≡ R
---------- (replacement 2)

$P \supset R$

$P \equiv Q$
---------- (symmetricity)
$Q \equiv P$

In Appendix 5, we show the screen layout of the proofs of the following properties of a factorial program :

Termination : $x \geq 0 \supset$ <z := 1 ; ((x >0)? ; z := x × z ; x := x - 1)* ; (x = 0)?>true
Partial Correctness : $x = n \supset$ [z := 1 ; ((x>0)? ; z := x × z ; x := x - 1)* ; (x = 0)?](z = n!)
Total Correctness : $x \geq 0 \wedge x = n \supset$ <z := 1 ; ((x>0)? ; z := x × z ; x := x - 1)* ; (x = 0)?>(z = n!)

Intensional logic, reflective proof and Montague's semantics ([Gallin 75])

The intensional logic is a higher-order modal logic based on the type theory, which requires context-sensitive constraints on terms. It includes a lot of complicated logical concepts which however are all well described within the framework of DCG and the rule description conventions.

Syntax definition in terms of DCG

Meta language :

```
meta-formula --> pred-const, "(", term, ")" ;
meta-formula --> meta-formula, meta-imply,  meta-formula ;
pred-const --> "beweis" ;
meta-imply --> "=>" ;
meta-variable(_) --> "X" | "Y" ;
meta-term(_) -->  "R" | "S" | "A" | "B" | "P" | "F" | "G" ;
meta-term(_) --> meta-variable(_) ;
meta-term(T) --> meta-term(_), colon, type(T) ;
meta-type(_) --> "a" | "b" | "c" ;
meta-type(_) --> "T" | "T1" | "T2" | "T3" ;
```

Object language (of IL) :

```
term(T2) --> term((s,(T1,T2))), left-brace, term(T1), right-brace ;
left-brace --> "{" ;
right-brace --> "}" ;
term(t) --> term(t), imply,  term1(t) ;
imply --> "⊃" ;
```

```
term(t) --> term1(t) ;
term1(t) --> term1(t), or,  term2(t) ;
or --> "∨" ;
term1(t) --> term2(t) ;
term2(t) --> term2(t), and, term3(t) ;
and --> "∧" ;
term2(t) --> term3(t) ;
term3(T) --> term3(T), equality, term4(T) ;
equality --> "=" ;
term3(T) --> term4(T) ;
term4(T2) --> term4((T1,T2)), apply, term5(T1) ;
apply --> "•" .
term4(T) --> term5(T) ;
term5(T) --> "(", term(T), ")" ;
term5(t) --> not, term5(t) ;
not --> "~" ;
term5(t) --> bind-op, variable(T), ".", term5(t) ;
bind-op --> "∀" ;
bind-op --> "∃" ;
term5((T1,T2)) --> lambda, variable(T1), ".", term5(T2) ;
lambda --> "λ" ;
term5((s,T)) --> intension, term5(T) ;
intension --> "^" ;
term5(T) --> extension› term5((s,T)) ;
extension --> "∨" ;
term5(t) --> necessary, term5(t) ;
necessary --> "[]" ;
term5(t) --> possible, term5(t) ;
possible --> "<>" ;
term5(T) --> variable(T) ;
term5(T) --> constant(T);
term5(T) --> meta-term1(T), "(", meta-term(_), ")" ;
meta-term1(T) --> meta-term(T) ;
term5(T) --> meta-term(T) ;
variable(T) --> var-sym, colon, type(T) ;
variable(T) --> meta-variable, colon, type(T) ;
constant(t) --> true, colon, t ;
true --> "true" ;
t --> "t" ;
```

constant(t) --> false, colon, t ;

false --> "false" ;

constant (T)--> const-sym, colon, type(T) ;

type(e) --> "e" ;

type(t) --> "t" ;

type((T1,T2)) --> "(", type(T1), comma, , type(T2), ")" ;

comma --> "," ;

type((s,T)) --> "(", s, comma, type(T), ")" ;

type(T) --> meta-type(_) ;

s --> "s" ;

var-sym --> "x" | "y" | "p" ;
const-sym --> "fish" | "believe" | "walk" | "j" ;
colon --> ":" ;

Axioms and Theorems

Axioms :

(1) G:(t,t) • true:t ∧ G:(t,t) • false:t = ∀X:t.G:(t,t)•X:t
(2) X:a = Y:a ⊃ F:(a,t)•X:a = F:(a,t)•Y:a
(3) ∀X:a.(F:(a,b)•X:a = G:(a,b)•X:a) = (F:(a,b) = G:(a,b))
(4) (λX:a. A(X:a))•B = A(B)
(5) □(ᵛF:(s,a) = ᵛG:(s,a)) = (F:(s,a) = G:(s,a))
(6) ᵛ^A:a = A:a

Theorems:

(1) (P:t = true:t) = P:t
(2) λX:a.Q:b = λX:a.Q:b

Inference rules

Meta-Rule :

$$\frac{\text{beweis}(A)}{A}\ \text{(Reflection-1)}$$

$$\frac{A}{\text{beweis}(A)}\ \text{(Reflection-2)}$$

$$\frac{\begin{array}{c}[A]\\ :\\ B\end{array}}{}$$

---------- (=>I)
A => B

Object-Rule (IL Rule) :

A(R) R = S
---------- (Replace-1)
A(S)

A(B) = A(R) R = S
---------- (Replace-2)
A(B) = A(S)

R = S
---------- (Symmetricity)
S = R

Rewriting rules

λX:a. P:t = λX:a. true:t
---------- (∀-Definition)
∀X:a. P:t

A{R}
---------- (Brace convention)
(^A)•R

F•G
---------- (Notational convention)
F(G)

The following metatheorem is ingeniously proved with the help of the reflection principle ([Weyhrauch 80]) whosse screen layout is shown in Appendix 6.

|- P:t => |- ∀x:a. P:t *(Generalization rule)*

In Montague's language theory, natural language sentences are first translated into expressions in intensional logic, which in turn are analyzed with the posible world semantics. With the defined intensional logic, the following intensional formula :

(λp:(s,(e,t)).∃x:e.(fish:(e,t)•x:e ∧ p:(s,(e,t)){x:e})) • ^λy:e.(believe:((s,t),(e,t)) •
^(walk:(e,t) • y:e) • j:e),

which is a translation of a natural language sentence "John believes that a fish walks", reduces to a more simple one :

$$\exists x{:}e.(\mathrm{fish}{:}(e,t)\bullet x{:}e \wedge \mathrm{believe}{:}((s,t),(e,t)) \bullet \hat{}(\mathrm{walk}{:}(e,t) \bullet x{:}e) \bullet j{:}e).$$

For other logical experiments, we will only list the theorems which were actually proved by using EUODHILOS.

First-order logic (with NK)

(1) Smullyan's logical puzzles (originally examples in combinatory logic) [Smullyan 85]

Axioms :

1. $\forall x\ m\bullet x = x\bullet x$ *(Mockingbird condition)*
2. $\forall x \forall y \exists z \forall w\ z\bullet w = x\bullet(y\bullet w)$ *(Composition)*

Theorems :

1. $\vdash \forall x \exists y(x\bullet y = y)$ *(Every bird of the forest is fond of at least one bird)*
2. $\vdash \exists x(x\bullet x = x)$ *(At least one bird is egocentric or narcissistic)*

(2) Unsolvability of the halting problem [Burkholder 87]

Premises :

1. $\exists x(A(x) \,\&\, \forall y(C(y) \supset \forall z D(x,y,z))) \supset \exists w(C(w) \,\&\, \forall y(C(y) \supset \forall z D(w,y,z)))$
   (Church's thesis)
2. $\forall w(C(w) \,\&\, \forall y(C(y) \supset \forall z D(w,y,z)) \supset \forall y \forall z((C(y) \,\&\, H(y,z) \supset H(w,y,z) \,\&O(w,g))$
   $\&\ (C(y) \,\&\, \sim H(y,z) \supset H(w,y,z) \,\&\, O(w,b))))$
3. $\exists w(C(w) \,\&\, \forall y((C(y) \,\&\, H(y,y) \supset H(w,y,y) \,\&\, O(w,g)) \,\&(C(y) \,\&\, \sim H(y,y) \supset$
   $H(w,y,y) \,\&\, O(w,b)))) \supset \exists v(C(v) \,\&\, \forall y((C(y) \,\&\, H(y,y) \supset H(v,y) \,\&\, O(v,g)) \,\&$
   $(C(y) \,\&\, \sim H(y,y) \supset H(v,y) \,\&\, O(v,b))))$
4. $\exists v(C(v) \,\&\, \forall y((C(y) \,\&\, H(y,y) \supset H(v,y) \,\&\, O(v,g)) \,\&\, (C(y) \,\&\, \sim H(y,y) \supset H(v,y) \,\&$
   $O(v,b))) \supset \exists u(C(u) \,\&\, \forall y((C(y) \,\&\, H(y,y) \supset \sim H(u,y)) \,\&\, (C(y) \,\&\, \sim H(y,y) \supset H(u,y)$
   $\&\ O(u,b)))$

Conclusion :

$\vdash \sim \exists x(A(x) \,\&\, \forall y(C(y) \supset \forall z D(x,y,z)))$

(no algorithm to solve the halting problem exists)

(3) Proof by structural induction on list [Eriksson 82]

$\vdash \forall x \forall y \forall z.\ \mathrm{append}(\mathrm{append}(x,y),z) = \mathrm{append}(x,\mathrm{append}(y,z))$

(associativity of append function)

Second-order logic and a simple equivalence proof

$$\forall P[P(0) \wedge \forall n(P(n) \supset P(n+1)) \supset \forall nP(n)] \equiv \forall R[\forall n(\forall j(j < n \supset R(j)) \supset R(n)) \supset \forall nR(n)]$$

(The principle of the mathematical induction is equivalent to the principle of the complete induction)

Propositional modal logic (T) and modal reasoning about programs ([Burstall 74])

$$\vdash \langle\rangle p \wedge [](p \supset q) \supset \langle\rangle(p \wedge q)$$

(A strong correctness assertion is implied from a termination assertion and a weak correctness assertion)

In the future we plan to attack a logic of knowledge and belief, various other logics of programs such as algorithmic logics based on infinitary logic, non-monotonic logic, relevant logic and so on.

RELATED WORKS

Much work has been devoted to building systems for checking and building formal proofs in various logical systems. A number of ways which may be used for assisting human reasoning, including automatic theorem proving, proof checker [de Bruijn 80][Weyhrauch 80] [Ketonen 84][Trybulec 85][Sawamura 86], proof constructor [Gordon 79][Constable 82][Constable 86] and general system for computer-aided reasoning [Coquand 85][Sawamura 87][Griffin 87][Harper 87][Minami 88][Satoh 88a], are comparatively examined in [Minami 88]. Here we are confined ourselves to various approaches to the general system for computer-assisted reasoning to which much attention have been recently paid. Let us briefly see only the distinction of a formal system description language in each approach since there have not yet been so much work as to the other aspects such as proving methodology in computer-assisted reasoning and reasoning-oriented human-computer interface, to such an extent that comparative studies become possible.

In [Sawamura 86], Prolog is employed as a logic description language as well as an implementation language of a proof constructor. In [Felty 88] and [Miller 87], λProlog, which is a higher-order version of Prolog and hence more expressive than Prolog, is proposed to specify theorem provers. In [Harper 87] and [Griffin 87], Martin-Löf's intuitionistic type theory is applied for building a logical framework (LF) which allows for a general treatment of syntax, inference rules, and proofs in terms of a typed λ–calculus with dependent types. It also has the advantage of a smooth treatment of discharge and variable occurrence conditions in rules. In [Reps 84], the axioms and inference rules of a formal logical system can be expressed

as productions and semantic equations of an attribute grammar. Then, dependencies among attributes, as defined in the semantic equations of such a grammar, express dependencies among parts of a proof. Among these works, those of [Harper 87], [Griffin 87] and [Reps 84] are closely related to our work. What they are aiming principally at seems to be automatic check of rule conditions basically in one way reasoning, with which we are confronted in applying a rule. In our approach, we have to attain it in the framework of our proving methodology, that is, in the environment that allows us to reason forward, reason backward, or reason in a mixture of them. The uniform treatment in this situation, however, is left open. In a current version of EUODHILOS, the problem of automatically checking the application conditions for rules is avoided by alerting to a user when he or she applies a rule with the conditions, in other words, a user is supposed to be responsible for applying the rules. Note that this is not our final solution to that. In [Gordon 82], the metalanguage (ML) for interactive proof in LCF [Gordon 79], a polymorphically typed, functional programming language, are used to show how logical calculi can be represented and manipulated within it. In [Abrial 84], constructing a general-purpose proof checker is undertaken through devising a theory of proofs. It is "general purpose" in that it may take as input the axiomatization of a formal theory together with a proof written within this theory. A theory of proofs is a kind of a specification language for formal system from the viewpoint of software engineering, and also a formal system description language. His approach is based on the rigorious approach to program construction : to define a theory and then to apply it.

In addition to such a purely theoretical interest as what a general theory of logics is, an important benefit of these treatments of formal systems is, although their approaches are different, that logic-independent tools for proof editors, proof chekers, and proof constructors can be constructed. As to logic-dependent tools, we think that it would be better to provide them by designing an appropriate metalanguage such as ML [Gordon 79].

Among these general-purpose reasoning assistant sytems, it seems fair to say that it is only our system EUODHILOS that incorporates such a distinctive feature as proving methodology plus logic defining capability, emphasizing visual interface for reasoning.

CONCLUDING REMARKS AND DIRECTIONS FOR FUTURE RESEARCH

We have presented the unique features of a general-purpose reasoning assistant system EUODHILOS which is under development. And also we have shown its advantages of our approach and potentials through a number of formal systems and their proof examples. Our conclusion of this paper is embodied in the following emblem :

$$\forall \text{Universe} \exists \text{Logic}.\ \text{EUODHILOS}(\text{Universe}, \text{Logic})$$

meaning that EUODHILOS helps to realize a logic relevent for a universe of discourse.

As a matter of fact, we have been confirmed in the following intended points by those experiments.

(i) Advantages of generality

The generality of EUODHILOS have been tested by using it to define various logics and to verify proofs expressed within them. Almost every logic together with its proof example was created in several hours. If we had have to develope a reasoning system with same functions as EUODHILOS for each logic from scratch, how much time would it have taken to do it ? And we would have to do another job in the almost same manner if we had needed other logic. Therefore we may conclude that EUODHILOS have demonstrated the usefulness of its generality in a much wider range of applications.

(ii) Definite clause grammar approach to the definition of logical syntax

Definite clause grammar is more natural and easier for users to define a logical syntax, compared with other approaches to logical system description languages mentioned above. And also it requires less knowledge to describe logics.A formula editor and a facility to test the defined language serve to check the intended syntax. It has been proved that these greatly lighten a user's burden in setting up his own language.

(iii) Proving methodology based on sheets of thought

Lots of experiments for proving have convinced us that reasoning by several sheets of thought naturally coincides with the ways of human thinking such as from the parts to the whole (typically found in the (Far) Eastern ideas) and from the whole to the parts (typically found in the Western ideas). It may be also expected that they turn out to give a promising way towards proving in large.

(iv) Visual interface

The visual interface for reasoning not only has been useful but also served to define the logics and to conceive ideas for constructing the proofs described in the previous Section.

An attempt of constructing a general-purpose reasoning assistant system is, however, only at the initial stage of research and development, and is lacking a number of significant issues which should be taken into consideration. We shall touch upon some of future research themes which may be helpful to augment and raise the current level of EUODHILOS.

(1) Augmentation of formal system description language

The current state of the formal system description language is defficient in some respects. Much efforts have to be payed on making the logic description language more expressive. For example, in the current framework, rule descriptions for tablaux method, some formulation of relevant logic, etc., seem not to be expressible. Furthermore, automatic mechanism for checking rule application conditions is not incorporated in EUODHILOS as remarked in the previous section. To overcome these deficiencies, we would need some more powerful rule description language and method.

(2) Investigation of higher-level supporting functions for reasoning

Developing a language for proof strategies, incorporating metatheory, etc., are important subjects since these could attain increasing the naturalness and efficiency of proofs.

(3) Maintaining a relational dependency among various theories

Various theories or logics are used to be involved in a proof. Let us consider the following situation : There exists a number of theories or logics together with various kinds of databases, they may be mutually dependent in the sense of the referential relations and we want to modify or revise a theory or underlying logic. Then what happens? Obviously, relational inconsistencies among theories may arise with such a modification and revision of theories or logics. The reader will notice that this is a kind of non-monotonic phenomenon.

(4) Opening up a new application field of reasoning by EUODHILOS

The unique features and potentials of EUODHILOS could suggest a new direction to CAI system for logics. We are now particularly interested in clarifying the feasibility of using EUODHILOS as a tool of logical model construction and specialized use of EUODHILOS such as a basis of computer-aided programming.

(5) Improvement and refinement of human-computer interface for the reasoning system

We have tried to analyze intrinsically how reasoning-oriented human-computer interface should be. However, it seems to lack a uniform and systematic point of view for such an interface in the present form.

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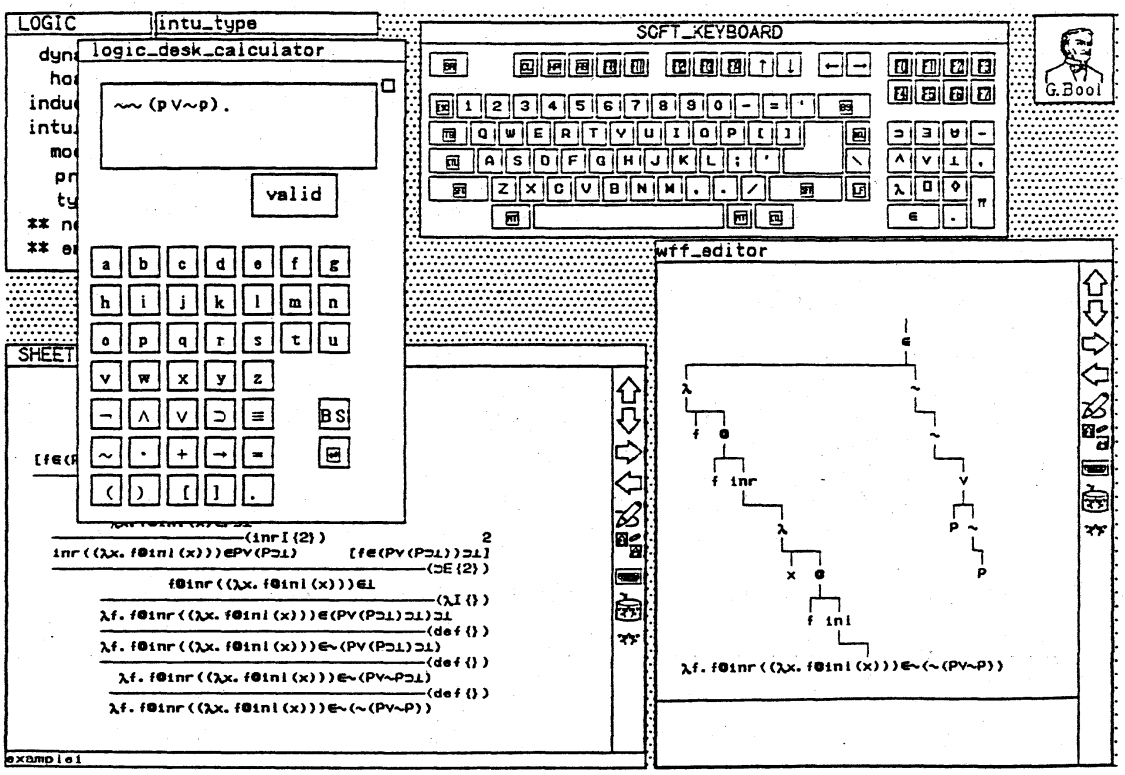
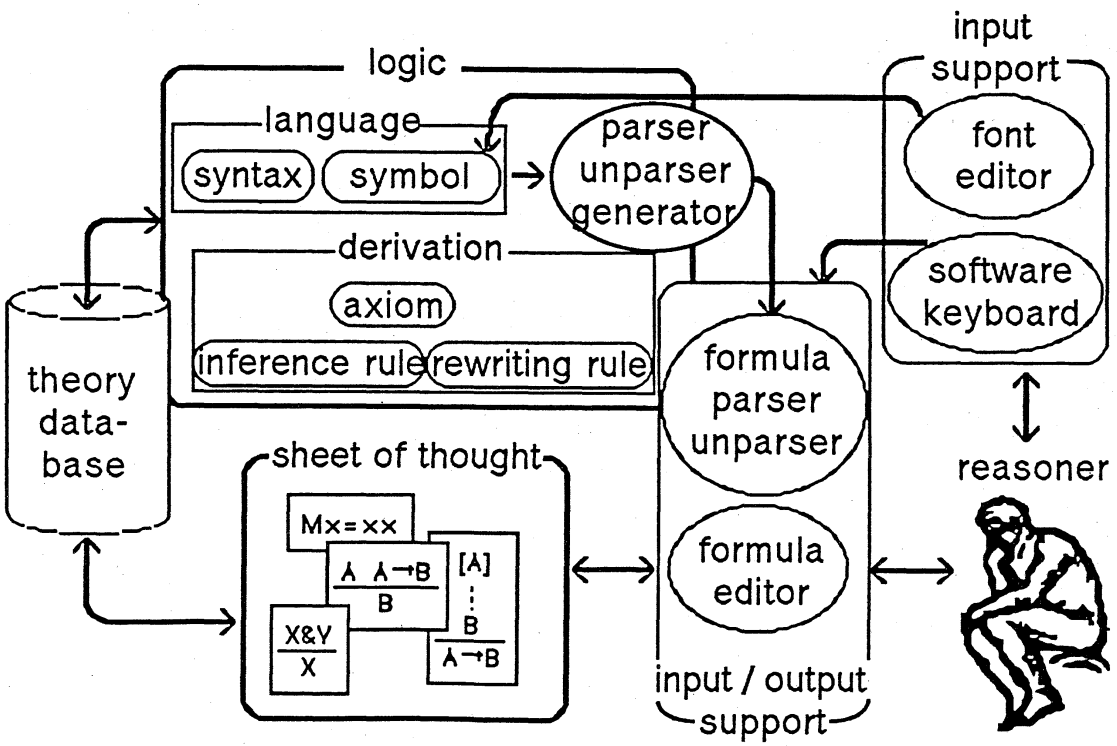
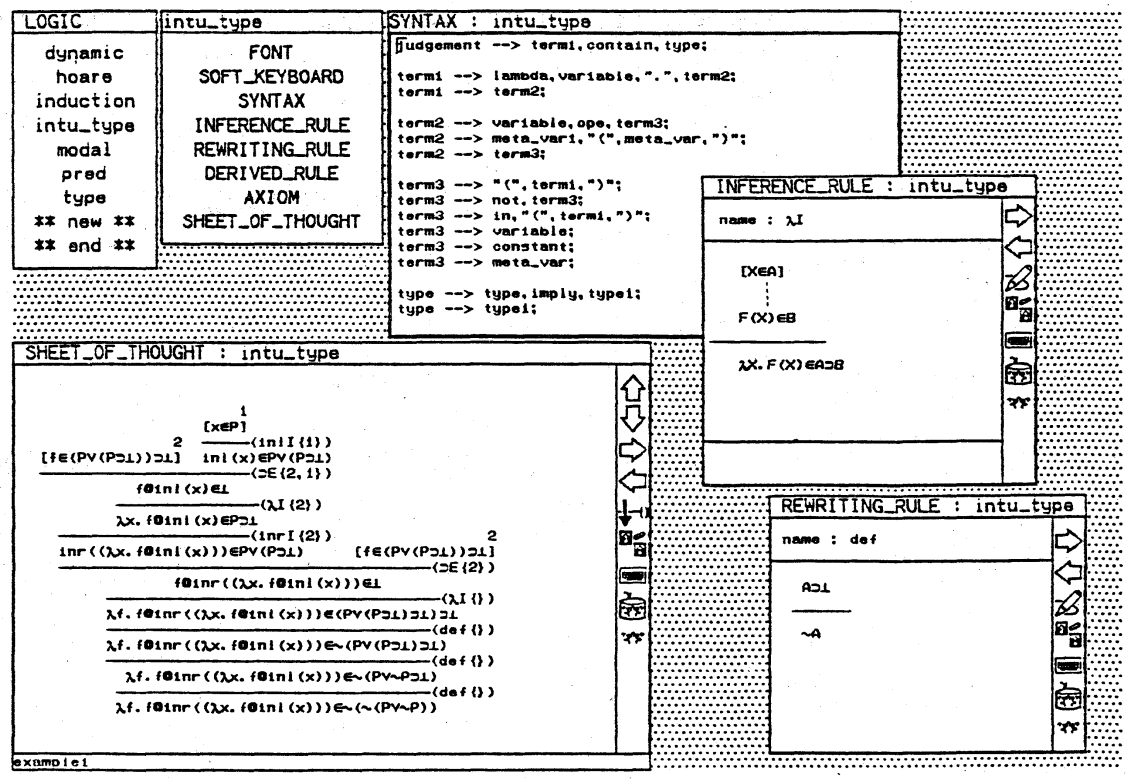
Appendix 1. Reasoning-oriented human-computer interface



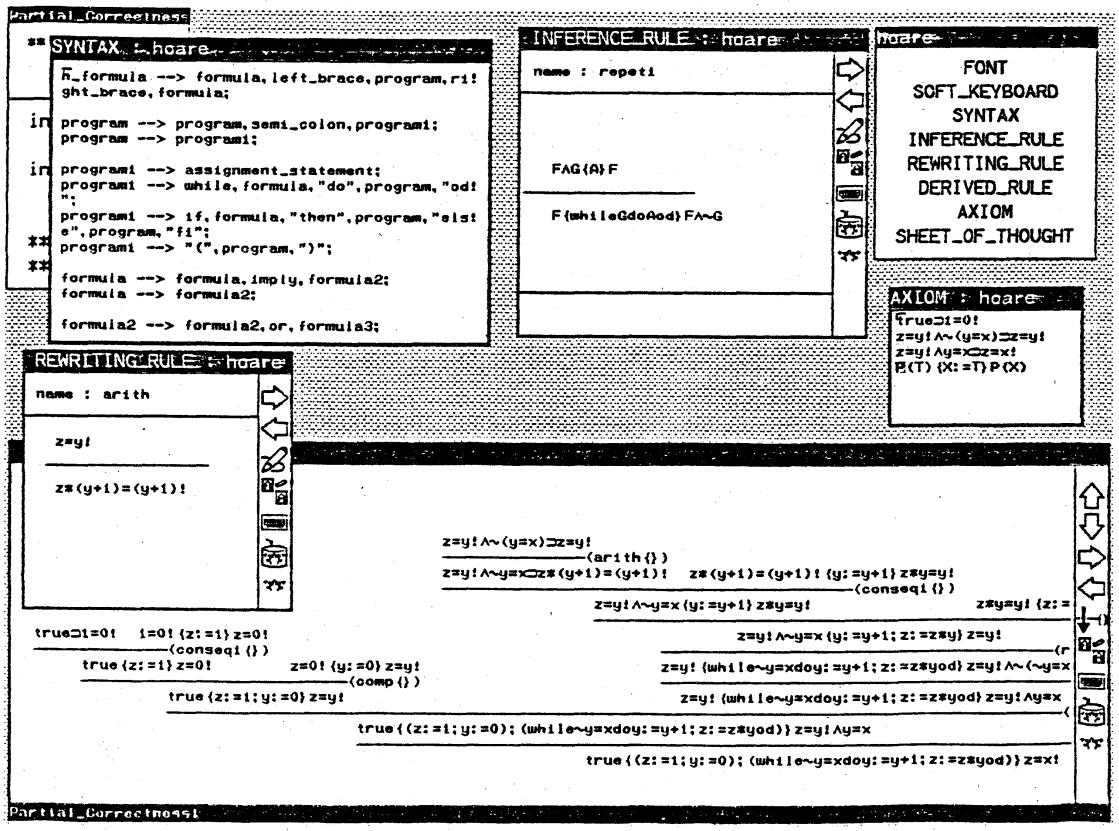
Appendix 2. An illustration of EUODHILOS



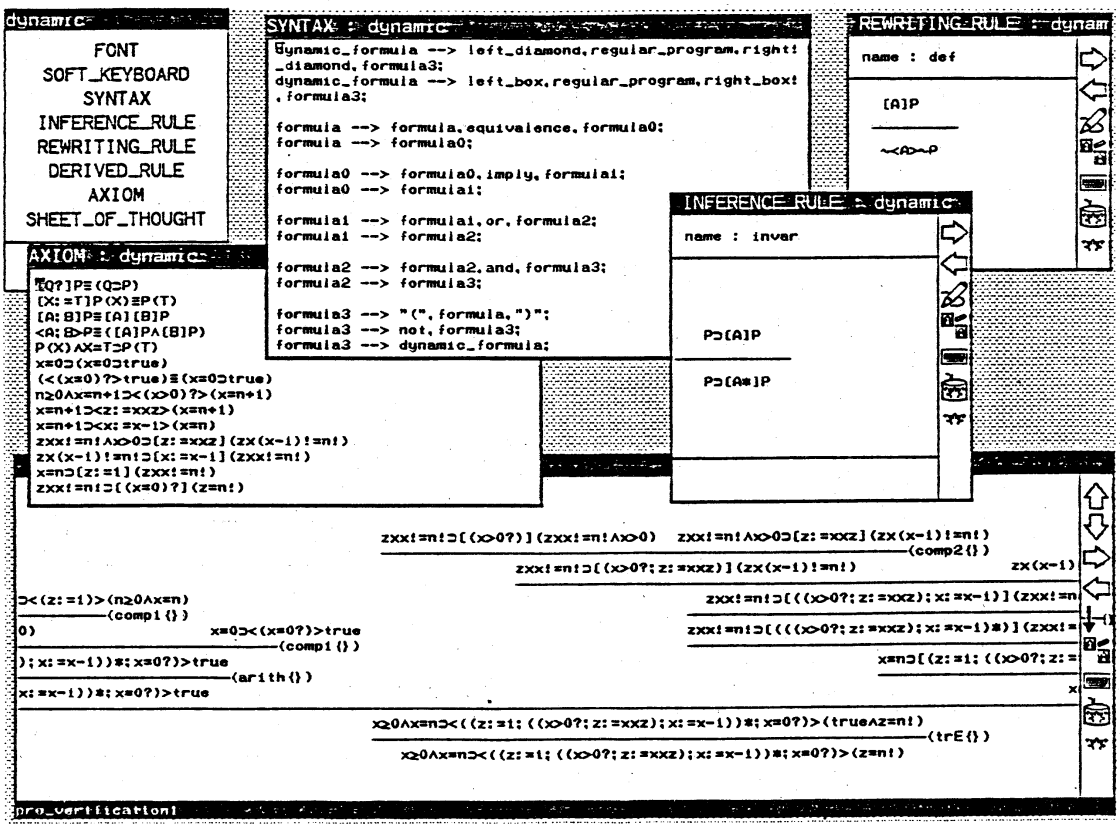
Appendix 3. Intuitionistic type theory and a constructive proof



Appendix 4. Hoare logic and correctness proof of a program



Appendix 5. Dynamic logic and reasoning about programs



Appendix 6. Intensional logic and a reflective proof